



## **ENERGY AND ENTROPY AS THE FUNDAMENTS OF THEORETICAL PHYSICS**

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### **ABSTRACT:**

Einstein's article titled, "The Fundaments of Theoretical Physics", from Science, Washington, D.C., May 24, 1940, is presented in its entirety as it is an outstanding presentation of the history and status of the foundations of theoretical physics as it stood in 1940. Further, it provides the background for discussing the new view of the fundaments of theoretical physics provided by the energy and entropy foundation of the Dynamic Theory.

**Keywords:** Energy, Entropy, Equations of motion, Quantum Mechanics, Gauge fields

### **INTRODUCTION**

Einstein spent virtually his entire working life in theoretical physics. He had an extremely clear view of what the foundations of theoretical physics was and should be. He was able to express this view so vividly that it is hard to imagine being able to improve upon his words. Here is the article, published in 1940, in which Einstein sets forth the fundaments of theoretical physics as he understood it then. Little has changed in the fundaments until recently. Following Einstein's article there is a brief discussion of more recent developments in the foundations of theoretical physics that display the

fundamental roles of energy and entropy in fundaments of theoretical physics.

Science is the attempt to make the chaotic diversity of our sense-experience correspond to a logically uniform system of thought. In this system single experiences must be correlated with the theoretic structure in such a way that the resulting coordination is unique and convincing.

The sense-experiences are the given subject-matter. But the theory that shall interpret them is manmade. It is the result of an extremely laborious process of adaptation: hypothetical, never completely final, always subject to question and doubt. The scientific way of forming concepts differs from that which we use in our daily life, not basically, but merely in the more precise definition of concepts and conclusions; more painstaking and systematic choice of experimental material; and greater logical economy. By this last we mean the effort to reduce all concepts and correlations to as few as possible logically independent basic concepts and axioms.

What we call physics comprises that group of natural sciences which base their concepts on measurements; and whose concepts and propositions lend themselves to mathematical formulation. Its realm is

accordingly defined as that part of the sum total of our knowledge which is capable of being expressed in mathematical terms. With the progress of science, the realm of physics has so expanded that it seems to be limited only by the limitations of the method itself.

The larger part of physical research is devoted to the development of the various branches of physics, in each of which the object is the theoretical understanding of more or less restricted fields of experience, and in each of which the laws and concepts remain as closely as possible related to experience. It is this department of science, with its ever-growing specialization, which has revolutionized practical life in the last centuries, and given birth to the possibility that man may at last be freed from the burden of physical toil.

On the other hand, from the very beginning there has always been present the attempt to find a unifying theoretical basis for all these single sciences, consisting of a minimum of concepts and fundamental relationships, from which all the concepts and relationships the single disciplines might be derived by logical process. This is what we mean by the search for a foundation of the whole of physics. The confident belief that this ultimate goal may be reached is the chief source of the passionate devotion which has always animated the researcher. It is in this sense that the following observations are devoted to the foundations of physics.

From what has been said it is clear that the word foundations in this connection does not mean something analogous in all respects to the foundations of a building. Logically considered, of course, the various single laws of physics rest upon this foundation. But whereas a building may be seriously damaged by a heavy storm or spring flood, yet its foundations

remain intact, in science the logical foundation is always in greater peril from new experiences or new knowledge than are the branch disciplines with their closer experimental contacts. In the connection of the foundation with all the single parts lies its great significance, but likewise its greatest danger in face of any new factor. When we realize this, we are led to wonder why the so-called revolutionary epochs of the science of physics have not more often and more completely changed its foundation than has actually been the case.

The first attempt to lay a uniform theoretical foundation was the work of Newton. In his system everything is reduced to the following concepts: (1) Mass points with invariable mass; (2) action at a distance between any pair of mass points; (3) law of motion for the mass point. There was not, strictly speaking, any all-embracing foundation, because an explicit law was formulated only for the actions-at-a-distance of gravitation; while for other actions-at-a-distance nothing was established a priori except the law of equality of action and reaction. Moreover, Newton himself fully realized that time and space were essential elements, as physically effective factors, of his system, if only by implication.

This Newtonian basis proved eminently fruitful and was regarded as final up to the end of the nineteenth century. It not only gave results for the movements of the heavenly bodies, down to the most minute details, but also furnished a theory of the mechanics of discrete and continuous masses, a simple explanation of the principle of the conservation of energy and a complete and brilliant theory of heat. The explanation of the facts of electrodynamics within the Newtonian system was more forced; least convincing of all, from the very beginning, was the theory of light.

It is not surprising that Newton would not listen to a wave theory of light; for such a theory was most unsuited to his theoretical foundation. The assumption that space was filled with a medium consisting of material points that propagated light waves without exhibiting any other mechanical properties must have seemed to him quite artificial. The strongest empirical arguments for the wave nature of light, fixed speeds of propagation, interference, diffraction, polarization were either unknown or else not known in any well-ordered synthesis. He was justified in sticking to his corpuscular theory of light.

During the nineteenth century the dispute was settled in favor of the wave theory; Yet no serious doubt of the mechanical foundation of physics arose, in the first place because nobody knew where to find a foundation of another sort. Only slowly, under the irresistible pressure of facts, there developed a new foundation of physics, field-physics.

From Newton's time on, the theory of action-at-a-distance was constantly found artificial. Efforts were not lacking to explain gravitation by a kinetic theory, that is, on the basis of collision forces of hypothetical mass particles. But the attempts were superficial and bore no fruit. The strange part played by space (or the inertial system) within the mechanical foundation was also clearly recognized, and criticized with especial clarity by Ernst Mach

The great change was brought about by Faraday, Maxwell, and Hertz, as a matter of fact halfunconsciously and against their will. All three of them, throughout their lives, considered themselves adherents of the mechanical theory. Hertz had found the simplest form of the equations of the electromagnetic field, and declared that any theory leading to these equations was

Maxwellian theory. Yet toward the end of his short life he wrote a paper in which he presented as the foundation of physics a mechanical theory freed from the force-concept.

For us, who took in Faraday's ideas so to speak with our mother's milk, it is hard to appreciate their greatness and audacity. Faraday must have grasped with unerring instinct the artificial nature of all attempts to refer electromagnetic phenomena to actions-at-a-distance between electric particles reacting on each other. How was each single iron filing among a lot scattered on a piece of paper to know of the single electric particles running round in a nearby conductor? All these electric particles together seemed to create in the surrounding space a condition which in turn produced a certain order in of the filings. These spatial states, today called fields, if their geometrical structure and interdependent action were once rightly grasped, would, he was convinced, furnish the clue to the mysterious electromagnetic

interactions. He conceived these in fields as states of mechanical stress in a space-filling medium, similar to the states of stress in an elastically distended body. For at that time this was the only way one could conceive of only states that were apparently continuously distributed in space. The peculiar type of mechanical interpretation of these fields remained in the background--a sort of placation of the scientific conscience in view of the mechanical tradition of Faraday's time. With the help of these new field concepts Faraday succeeded in forming a qualitative concept of the whole complex of electromagnetic effects discovered by him and his predecessors. The precise formulation of the timespace laws of those fields was the work of Maxwell. Imagine his feelings when the differential equations

he had formulated proved to him that electromagnetic fields spread in the form of polarized waves and with the speed of light! To few men in the world has such an experience been vouchsafed. At that thrilling moment he surely never guessed that the riddling nature of light, apparently so completely solved, would continue to baffle succeeding generations. Meantime, it took physicists some decades to grasp the full significance of Maxwell's discovery, so bold was the leap that his genius forced upon the conceptions of his fellow-workers. Only after Hertz had demonstrated experimentally the existence of Maxwell's electromagnetic waves did resistance to the new theory break down.

But if the electromagnetic field could exist as a wave independent of the material source, then the electrostatic interaction could no longer be explained as action-at-a-distance. And what was true for electrical action could not be denied particles for gravitation. Everywhere Newton's actions-at-a-distance gave way to fields spreading with finite velocity.

Of Newton's foundation there now remained only the material mass points subject to the law of motion. But J. J. Thomson pointed out that an electrically charged body in motion must, according to Maxwell's theory, possess a magnetic field whose energy acted precisely as does an increase of kinetic energy to the body. If, then, a part of kinetic energy consists of field energy, might that not then be true of the whole of the kinetic energy? Perhaps the basic property of matter, its inertia, could be explained within the field theory? The question led to the problem of an interpretation of matter in terms of field theory, the solution of which would furnish an explanation of the atomic structure of matter. It was soon realized that Maxwell's theory could not accomplish such a

program. Since then many scientists have zealously sought to complete the field theory by some generalization that should comprise a theory of matter; but so far such efforts have not been crowned with success. In order to construct a theory, it is not enough to have a clear conception of the goal. One must also have a formal point of view which will sufficiently restrict the unlimited variety of possibilities. So far this has not been found; accordingly the field theory has not succeeded in furnishing a foundation for the whole of physics.

For several decades most physicists clung to the conviction that a mechanical substructure would be found for Maxwell's theory. But the unsatisfactory results of their efforts led to gradual acceptance of the new field concepts as irreducible fundamentals--in other words, physicists resigned themselves to giving up the idea of a mechanical foundation.

Thus physicists held to a field-theory program. But it could not be called a foundation, since nobody could tell whether a consistent field theory could ever explain on the one hand gravitation, on the other hand the elementary components of matter. In this state of affairs it was necessary to think of material particles as mass points subject to Newton's laws of motion. This was the procedure of Lorentz in creating his electron theory and the theory of the electromagnetic phenomena of moving bodies.

Such was the point at which fundamental conceptions had arrived at the turn of the century. Immense progress was made in the theoretical penetration and understanding of whole groups of new phenomena; but the establishment of a unified foundation for physics seemed remote indeed. And this state of things has

even been aggravated by subsequent developments. The development during the present century is characterized by two theoretical systems essentially independent of each other: the theory of relativity and the quantum theory. The two systems do not directly contradict each other; but they seem little adapted to fusion into one unified theory. We must briefly discuss the basic idea of these two systems.

The theory of relativity arose out of efforts to improve, with reference to logical economy, the foundation of physics as it existed at the turn of the century. The so-called special or restricted relativity theory is based on the fact that Maxwell's equations (and thus the law of propagation of light in empty space) are converted into equations of the same form, when they undergo Lorentz transformation. This formal property of the Maxwell equations is supplemented by our fairly secure empirical knowledge that the laws of physics are the same with respect to all inertial systems. This leads to the result that the Lorentz transformation--applied to space and time coordinates--must govern the transition from one inertial system to any other. The content of the restricted relativity theory can accordingly be summarized in one sentence: all natural laws must be so conditioned that they are covariant with respect to Lorentz transformations. >From this it follows that the simultaneity of two distant events is not an invariant concept and that the dimensions of rigid bodies and the speed of clocks depend upon their state of motion. A further consequence was a modification of Newton's law of motion in cases where the speed of a given body was not small compared with the speed of light. There followed also the principle of the equivalence of mass and energy, with the laws of conservation of mass and energy becoming one and the same. Once

it was shown that simultaneity was relative and depended on the frame of reference, every possibility of retaining actions-at-a-distance within the foundation of physics disappeared, since that concept presupposed the absolute character of simultaneity (it must be possible to state the location of the two interacting mass points "at the same time").

The general theory of relativity owes its origin to the attempt to explain a fact known since Galileo's and Newton's time but hitherto eluding all theoretical interpretation: the inertia and the weight of a body, in themselves two entirely distinct things, are measured by one and the same constant, the mass. From this correspondence follows that it is impossible to discover by experiment whether a given system of coordinates is accelerated, or whether its motion is straight and uniform and the observed effects are due to a gravitational field (this is the equivalence principle of the general relativity theory). It shatters the concepts of the inertial system, as soon as gravitation enters in. It may be remarked here that the inertial system is a weak point of the Galilean-Newtonian mechanics. For there is presupposed a mysterious property of physical space, conditioning the kind of coordinate systems for which the law of inertia and the Newtonian law of motion hold good.

These difficulties can be avoided by the following postulate: natural laws are to be formulated in such a way that their form is identical for coordinate systems of any kind of states of motion. To accomplish this is the task of the general theory of relativity. On the other hand, we deduce from the restricted theory the existence of a Riemannian metric within the time-space continuum, which, according to the equivalence principle, describes both the

gravitational field and the metric properties of space. Assuming that the field equations of gravitation are of the second differential order, the field law is clearly determined.

Aside from this result, the theory frees field physics from the disability it suffered from, in common with the Newtonian mechanics, of ascribing to space those independent physical properties which heretofore had been concealed by the use of an inertial system. But it cannot be claimed that those parts of the general relativity theory which can today be regarded as final have furnished physics with a complete and satisfactory foundation. In the first place, the total field appears in it to be composed of two logically unconnected parts, the gravitational and the electromagnetic. And in the second place, this theory, like the earlier field theories, has not up till now supplied an explanation of the atomistic structure of matter. This failure has probably some connection with the fact that so far it has contributed nothing to the understanding of quantum phenomena. To take in these phenomena, physicists have been driven to the adoption of entirely new methods, the basic characteristics of which we shall now discuss.

In the year nineteen hundred, in the course of a purely theoretic investigation, Max Planck made a very remarkable discovery: the law of radiation of bodies as a function of temperature could not be derived solely from the laws of Maxwellian electrodynamics. To arrive at results consistent with the relevant experiments, radiation of a given frequency had to be treated as though it consisted of energy atoms of the individual energy  $h\nu$ , where  $h$  is Planck's universal constant. During the years following, it was shown that light was everywhere produced and absorbed in

such energy quanta. In particular Niels Bohr was able largely to understand the structure of the atom, on the assumption that atoms can have only discrete energy values, and that the discontinuous transitions between them are connected with the emission or absorption of such an energy quantum. This threw some light on the fact that in their gaseous state elements and their compounds radiate and absorb only light of certain sharply defined frequencies. All this was quite inexplicable within the frame of the hitherto existing theories. It was clear that at least in the field of atomistic phenomena the character of everything that happens is determined by discrete states and by apparently discontinuous transitions between them, Planck's Constant  $h$  playing a decisive role. The next step was taken by de Broglie. He asked himself how the discrete states could be understood by the aid of the current concepts, and hit on a parallel with stationary waves, as for instance in the case of the proper frequencies of organ pipes and strings in acoustics. True, wave actions of the kind here required were unknown; but they could be constructed, and their mathematical laws formulated, employing Planck's constant  $h$ . De Broglie conceived an electron revolving about the atomic nucleus as being connected with such a hypothetical wave train, and made intelligible to some extent the discrete character of Bohr's "permitted" paths by the stationary character of the corresponding waves.

Now in mechanics the motion of material points is determined by the forces or fields of force acting upon them. Hence it was to be expected that those fields of force would also influence de Broglie's wave fields in an analogous way. Erwin Schrödinger showed how this influence was to be taken into account, re-interpreting by an ingenious method

certain formulations of classical mechanics. He even succeeded in expanding the wave mechanical theory to a point where without the introduction of any additional hypotheses, it became applicable to any mechanical system consisting of an arbitrary number of mass points, that is to say possessing an arbitrary number of degrees of freedom. This was possible because a mechanical system consisting of  $n$  mass points is mathematically equivalent to a considerable degree to one single mass point moving in a space of  $3n$  dimensions.

On the basis of this theory there was obtained a surprisingly good representation of an immense variety of facts which otherwise appeared entirely incomprehensible. But on one point, curiously enough, there was failure: it proved impossible to associate with these Schrödinger waves definite motions of the mass points-and that, after all, had been the original purpose of the whole construction.

The quantum theoretical treatment of this case is as follows: at the time to we have a Schrödinger wave system entirely inside  $G$ . But from the time to onwards, the waves leave the interior of  $G$  in all directions, in such a way that the amplitude of the outgoing wave is small compared to the initial amplitude of the wave system inside  $G$ . The further these outside waves spread, the more the amplitude of the waves inside  $G$  diminishes, and correspondingly the intensity of the later waves issuing from  $G$ . Only after infinite time has passed is the wave supply inside  $G$  exhausted, while the outside wave has spread over an ever-increasing space.

All other cases are analogous. The aim of the theory is to determine the probability of the results of measurement upon a

system at a given time. On the other hand, it makes no attempt to give a mathematical representation of what is actually present or goes on in space and time. On this point the quantum theory of today differs fundamentally from all previous theories of physics, mechanistic as well as field theories. Instead of a model description of actual space-time events, it gives the probability distributions for possible measurements as functions of time.

It must be admitted that the new theoretical conception owes its origin not to any flight of fancy but to the compelling force of the facts of experience. All attempts to represent the particle and wave features displayed in the phenomena of light and matter, by direct recourse to a space-time model, have so far ended in failure. And Heisenberg has convincingly shown, from an empirical point of view, that any decision as to a rigorously deterministic structure of nature is definitely ruled out, because of the atomistic structure of our experimental apparatus. Thus it is probably out of the question that any future knowledge can compel physics again to relinquish our present statistical theoretical foundation in favor of a deterministic one which would deal directly with physical reality. Logically the problem seems to offer two possibilities, between which we are in principle given a choice. In the end the choice will be made according to which kind of description yields the formulation of the simplest foundation, logically speaking. At the present, we are quite without any deterministic theory directly describing the events themselves and in consonance with the facts.

For the time being, we have to admit that we do not possess any general theoretical basis for physics, which can be regarded as its logical foundation. The field theory, so

far, has failed in the molecular sphere. It is agreed on all hands that the only principle which could serve as the basis of quantum theory would be one that constituted a translation of the field theory into the scheme of quantum statistics. Whether this will actually come about in a satisfactory manner, nobody can venture to say .

Some physicists, among them myself, cannot believe that we must abandon, actually and forever, the idea of direct representation of physical reality in space and time; or that we must accept the view that events in nature are analogous to a game of chance. It is open to every man to choose the direction of his striving; and also every man may draw comfort from Lessing's fine saying, that the search for truth is more precious than its possession.

## DISCUSSION

The last paragraph states Einstein's lifelong belief that quantum mechanics should not ultimately form the foundations of physics. Today it is difficult to find a physicist publishing such a belief. Such is the belief in the fundamental nature of quantum mechanics. The success of the predictions of quantum mechanics and the vast growth of experimental data throughout the 20th century only adds to this conviction. A further impediment to looking into the foundations of physics is provided by the various branches of physics and the increased degree of specialization that exists today.

Einstein was not afraid of thinking thoughts not previously held. Yet when he contributed so much to the beginnings of quantum mechanics, those who pursued quantum mechanics as a fundamental basis for physics felt they had lost a leader when Einstein steadfastly refused to follow their path. It is now possible to show how correct he was in maintaining his stand

with the same rigorous logic that Einstein demanded of himself. There does indeed exist a simple set of fundamental postulates from which it may be shown that the basis of all the various branches of physics are but subsets of the totality of their description.

The starting point of this new line of thinking is so improbable as to be easily overlooked and yet it is the only foundation that has never been seen to offer predictions that differ from experience. This starting point is the laws of classical thermodynamics!

There are at least two reasons that classical thermodynamics would not be expected to provide such a foundation. First, thermodynamics, as currently studied, does not provide a description of motion like the mechanistic theories do. Secondly, texts teach, as Einstein believed, that classical thermodynamics might be obtained from statistical procedures applied to Newtonian mechanics.

The key insight needed to understand the fundamental nature of the laws of thermodynamics is to note that the first law is a Pfaff differential equation and to apply the second law of thermodynamics as Carathéodory did in 1909 [1]. Carathéodory's principle guarantees the existence of a property called entropy along with the energy statement of the first law. The form of these laws are such that they may be expressed, without preference, in any coordinate system and of any dimension, as Einstein stated should be required of a fundamental set of laws. Though the necessary, complimentary existence of energy and entropy appears to complicate any mechanistic description of nature, it is their simultaneous existence that provides a logical description.

Today, the concept of entropy is almost universally related to order or information. However, the concept demanded by the second law is best thought of as 'energy that becomes unavailable' as the thermal engineers have been known to call it. In this form, it is easier to connect the second law with the denial of perpetual motion. The more you do- the greater the amount of energy that becomes unavailable. This becomes the entropy principle for isolated systems. For all other systems, it requires the minimum free energy principle. This provides variational principles that may be used to determine motion should a geometric metric also be given.

In 1922, however, Schrödinger [7] noticed that, should one require a unity scale in a Weyl space, then only Bohr's quantized paths were allowed. Schrödinger went on to develop his wave equations of quantum mechanics in 1926 [8]. In 1927, London showed that the requirement of unity scale in a Weyl space could only be satisfied by paths that obeyed Schrödinger's wave equations [9]. Further, London showed that Schrödinger's wave function was proportional to Weyl's scale factor. Weyl seized upon this result and raised London's result to the level of a principle [10], referred to as Weyl's quantum principle [11]. This, together with Weyl's display that the gauge potentials formed the scale factor in his geometry, led to the electromagnetic gauge fields. Providing the basis for all the subsequent gauge fieldwork that Einstein referred to in 1940 and the work that has followed in the search for a description of the weak and the strong nuclear forces [11].

The fundamental laws require the quantization of gravitational phenomena for isentropic systems as well as a non-singular gravitational potential [12]. The appearance of the non-singular

gravitational potential changes the interpretation of black holes, the big bang, and red shifts of cosmological objects [14]. Now the tie between gravitation and quantum mechanics has been established.

One last feature of these fundamental laws should be mentioned. It concerns Einstein's position that two separate theoretical descriptions of light, on the one hand as a particle and on the other hand as a wave, was intolerable. Electromagnetic waves follow from Weyl's gauge fields; that is, from the Maxwell equations. Isentropic propagation of electromagnetic energy must also satisfy Weyl's quantum condition and hence, must simultaneously satisfy the wave equations and be quantized. Further, the fundamental laws require that the quantized, isentropic propagation of electromagnetic energy must satisfy Plank's blackbody radiation law [12]. The wave and the particle nature of light are, therefore, both required by these fundamental laws.

Einstein stated that there appears to be two choices for a foundation for physics; statistical or deterministic. Here we see a foundation that is fundamentally deterministic. Non-isolated systems and systems with variable entropy must be deterministic while isentropic systems must be quantized and, therefore, may have a statistical nature even though the probabilistic interpretation of Schrödinger's waves was shown by London to be in error. Einstein's desire for a logically simple foundation for physics is also satisfied; for these laws have been shown to produce the foundations of each of the various branches of physics without yet coming upon a measured difference from experiment.

The five dimensional wave equations require the transverse waves to consist of an electric, a magnetic and a gravitational

component rather than just the electric and magnetic components. This leads to the prediction that the electromagnetic energy density be non-zero when the radiation pressure vanishes. This suggests two things. First, since it is difficult to imagine the universe supporting a nonzero radiation pressure, then there must be a non-zero electromagnetic energy throughout the universe as is being measured. Secondly, this provides a new view of the zero point vacuum energy that may be more receptive to an engineering approach to mining it.

Another way new fundaments of theoretical physics may have an impact upon humans is to provide new logical basis upon which to look at our universe. This can lead to new understandings of known phenomena or to exciting predictions of new physics. For example, the study of the energy radiating from a blackbody led Planck to the first assumption of quanta and the first successful equation of quantum mechanics. What of the study of the blackbody itself? Obviously, a system radiating energy should not be considered to be isolated. Non-isolated systems have not been discussed above where the concentration was on isolated systems. An electron under the accelerating influence of a force that radiates energy is an example of a non-isolated system. So is a blackbody. The new fundaments of theoretical physics provides a variational principle in the minimum free energy principle and this principle should provide the equations of motion for these systems.

So much to learn, but so little time.

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